Renaud SIRDEY

A Gentle Introduction to the Deriche Optimal Edge Detector

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Introduction

The goal of this document is to introduce the Deriche edge detector, its reccursive implementation, and to summarize the pipeline of contours extraction. Several articles and reports gives more precise informations (see [Der] for example) but this document should (I hope) be sufficient for practical purpose.

1 The Deriche operator

1.1 A step edge model

The basic idea behind optimal operators is based on a continuous edge model which consists in a step signal corrupted by a gaussian white noise

$$I(x) = Au_{-1}(x) + n(x)$$

where $u_{-1}(x)$ denotes the unit-step function and $n(x)$ is a noise term whose mathematical expectation and variance are respectively 0 and σ^2.
We then consider the convolution $\Theta(x)$ of this signal with an edge detector $f(x)$

$$\Theta(x) = \int_{-\infty}^{+\infty} I(y)f(x-y)dy$$

1.2 The Canny criteria

According to the previous model Canny has proposed to find an optimal edge detection operator using the following three requirements :

1. Low probability of error (failing to mark of falsely marking real edge points). This criteria consists in finding an asymetric operator which maximizes the signal to noise ratio. Formally,

$$\Sigma = \frac{A\int_{-\infty}^{0} f(x)dx}{\sigma\sqrt{\int_{-\infty}^{+\infty} f^2(x)dx}}$$

2. Good localization: points marked as edges should be as close as possible to the true edge. The localization is defined as being the inverse of the standard deviation of the position of the true edge or formally,

$$\Lambda = \frac{A|f'(0)|}{\sigma\sqrt{\int_{-\infty}^{+\infty} f'^2(x)dx}}$$

3. Only one response to a single edge: consists in a constraint on the average distance between two local maxima ($x_{\max}$),

$$x_{\max} = \sqrt{\frac{\int_{-\infty}^{+\infty} f'^2(x)dx}{\int_{-\infty}^{+\infty} f''^2(x)dx}}$$

Canny has then proposed a FIR operator which maximizes the product $\Sigma\Lambda$ under the constraint that the third criterion is fixed to a constant value k. In practice the Canny operator is approximated by a Gauss function which leads to $\Sigma\Lambda = 0.92$.

1.3 The Deriche operator

Deriche [Der89] has then proposed an IIR operator which optimizes the Canny criteria, this operator has the following form

$$f(x) = \frac{S}{\omega} e^{-\alpha|x|} \sin \omega x$$

It is shown that the most efficient operator is the limit of the previous one when ω tends to 0. Therefore,

$$f(x) = Sxe^{-\alpha|x|}$$

This filter has a performance $\Sigma\Lambda$ equal to 2 (with $k = .44$) which is approximately 25% better than the IIR solution. An interesting property of this operator comes form the α parameter which allows to adapt it to the content of the images. Roughly (see [Der] page 25) if the image is noisy the α parameter has to be chosen relatively small (0.25 to 0.5) which means that Σ (detection) is favoured to the detriment of Λ (localisation). For a clean image α is chosen relatively large (≈ 1.0).

2 Implementation of the 2D Deriche operator

In [Der] (page 17) it is shown that the one-dimensional Deriche operator corresponds to two stable second order reccurive filters (this proof is based on computing the Z transform of the discrete IIR operator). This results (see [Der] again) can be extended to the two-dimensionnal operator. Here, we are going to focus on the implementation of the 2 dimensionnal Deriche filter, for more precise information [Der] (french readers) and [Der89] are highly recommended.

The first step in the computation of the filter consist in processing the rows of the image according to the following scheme (x is the input image)

$$y^{(1)}_{i,j} = a_1 x_{i,j} + a_2 x_{i,j-1} + b_1 y^{(1)}_{i,j-1} + b_2 y^{(1)}_{i,j-2} \tag{1}$$

going from left to right, and

$$y^{(2)}_{i,j} = a_3 x_{i,j+1} + a_4 x_{i,j+2} + b_1 y^{(2)}_{i,j+1} + b_2 y^{(2)}_{i,j+2} \tag{2}$$

going from right to left. This operation gives a temporary result, say λ, whose expression is

$$\lambda_{i,j} = c_1 (y^{(1)}_{i,j} + y^{(2)}_{i,j}) \tag{3}$$

The second step concerns the columns of λ

$$y^{(1)}_{i,j} = a_5 \lambda_{i,j} + a_6 \lambda_{i-1,j} + b_1 y^{(1)}_{i-1,j} + b_2 y^{(1)}_{i-2,j} \tag{4}$$

from top to bottom, and

$$y^{(1)}_{i,j} = a_7 \lambda_{i+1,j} + a_8 \lambda_{i+2,j} + b_1 y^{(1)}_{i+1,j} + b_2 y^{(1)}_{i+2,j} \tag{5}$$

from bottom to top. The final result is then obtained by

$$\Theta_{i,j} = c_2 (y^{(1)}_{i,j} + y^{(2)}_{i,j}) \tag{6}$$

The following set of parameters is used to compute the horizontal component of the gradient

$$a_1 = 0,\ a_2 = 1,\ a_3 = -1,\ a_4 = 0,\ c_1 = -(1 - e^{-\alpha})^2$$
$$a_5 = k,\ a_6 = ke^{-\alpha}(\alpha - 1),\ a_7 = ke^{-\alpha}(\alpha + 1),\ a_8 = -ke^{-2\alpha}, c_2 = 1$$

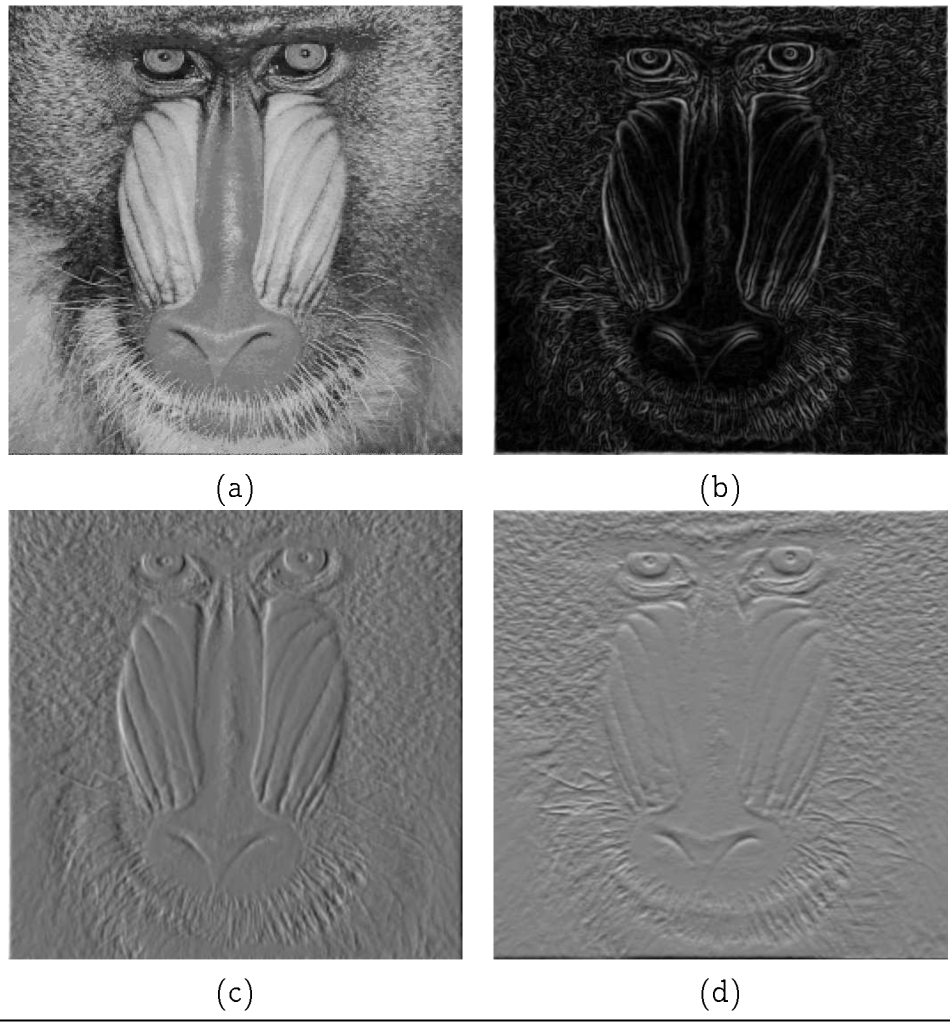
The vertical component can be obtain by swapping a_i and a_{i+4} and c_1 and c_2. For both components we have

$$k = \frac{(1 - e^{-\alpha})^2}{1 + 2\alpha e^{-\alpha} - e^{-2\alpha}},\ b_1 = 2e^{-\alpha},\ b_2 = -e^{-2\alpha}$$

Another set of parameters allows to compute the laplacian of the image but the most efficient way to perform features extraction is the gradient based approach[1]. Since the processing of a row (resp. a column) is independant of the computations required for other rows (resp. columns) it is possible to implement the Deriche operator on a parallel computer (see [LL96] for example). Figure 1 shows some results.

[1]Recall that differentiating corresponds to an operation of high-pass filtering and therefore amplifies the noise present in the image. Moreover, the gradient approach gives some information about contours orientation which is very useful (see page 7).

Figure 1 (a) original image (`singe.pgm`, 512×512), (b) gradient norm ($\alpha = 1.0$), (c) horizontal component, (d) vertical component.



3 Contours extraction

The goal of this document is not to focus on contours extraction, this section is just a short introduction to the problem of extracting contours in an image given an estimation of the gradient norm and of the partial derivatives.

3.1 Extraction of the local maxima

The first step, in order to extract thin contours i.e. contours of thickness equal to one pixel, is to extract the local maxima of the gradient norm in the gradient direction given by the vector $\left(\frac{\partial I}{\partial x}\ \frac{\partial I}{\partial y}\right)$. However, the gradient direction does not coincide (in general) to integer pixel coordinates. There are two common ways of dealing with this problem:

1. Take the gradient norm at the nearest neighbour of the pair of non-integer coordinates.

2. Perform a (bi)linear interpolation in order to obtain a value for the gradient norm at the non-integer coordinates given by the gradient direction (this is the method implemented here).

Note that both solutions have no *physical* meaning.

3.2 Hysteresis thresholding

The hysteresis thresholding method consist in eliminating the local maxima which are not significant according to the following rules. Given a high and a low threshold level we eliminate the local maxima whose gradient norm are less than the low level or less than the high level and not connected to a local maximum whose gradient norm is greater than the high threshold level.

Définition 1 (connectivity) *In a 8-connexity sense a local maximum is said to be connected to a local maximum whose gradient norm is greater than the high threshold level if one of its 8 nearest neighbours has a gradient norm greater than the high threshold level or if one of its neighbours is connected to a local maximum whose gradient norm is greater to the high threshold level.*

This thresholding technique gives better results than a one level threshold because it takes into account that some contours can be "hidden" in the noise. Figure 2 shows some results (see also figure 1).

Figure 2 Contours extracted after an hysteresis thresholding of the local maxima map (low=15, high=35).

References

[Der] R. Deriche. Techniques d'extraction de contours. www.inria.fr/robotvis/personnel/der/Demo/Features/fea% -tures.html.

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[LL96] J. Lakhal and L. Litzer. A parallelization of the deriche filter: A theoretical study and an implementation on the mapstar system. In Proceedings of the International Conference on Parallel Computing and Distributed Processing Techniques, page 867, 1996.